**3-D co-geometry 2**

**1.** Given the equation of a straight line is given as .

**(a)** Determine the vector equation of the straight line.

**(b)** Hence, find the coordinates of a point Q that lies on the straight line such that .

**(a)**

The vector equation is .

**(b)** Let be a point on the straight line.

or .

**2.** Find the Cartesian equation of the plane passing through the points and and containing the line : .

Direction of the given line is

Normal of the required plane is

Together with point , the equation of the required plane is

or

**3.** The position vectors of the points P, Q and R are respectively.

Let the plane π contains the points P, Q and R.

**(a)** Find a vector which is perpendicular to plane π.

**(b)** Find the area of .

**(c)** Obtain the Cartesian equation of plane π.

**(a)**

, which is a vector perpendicular to plane π.

**(b)** Area of =

**(c)** The normal of the plane is and is on the plane.

**4.** Find the vector normal to the plane .

Write down a vector normal to the plane . Show that is normal to both and . Given that the point lies on both and .

Write down the equation of line of intersection of and in the form of where t is a parameter.

The two directional vectors are on the plane, hence the normal is

, thus is normal to both and .

is a line parallel to the line of intersection of and . Since lies on both and , it must line on the intersection line.

Therefore, the equation of intersection line is .

**5.** .

Point P divides the line AC in the ratio 2 : 1 internally.

**(a)** Show that ABCD is a parallelogram.

**(b)** Calculate the exact area of the parallelogram ABCD.

**(c)** Find the position vector P and the angle APB in degrees corrected to one decimal place.

**(a)**

Since , the opposite sides are equal and parallel, therefore ABCD is a parallelogram.

**(b)**

Area of the parallelogram ABCD

= .

**(c)**

**6.** Show that the lines with equations

and intersect,

and find the position vector of their point of intersection.

The lines intersect if

Since (4), (5) satisfy (1), equations (1) – (3) has a unique solution : .

Therefore, the two given lines intersect.

The point of intersection is or in Cartesian form (1, 1, 1).

**7.** Given a sphere

**(i)**  Find the equations of the tangent planes to the sphere when x = 1 and y = 10.

**(ii)** Find a point on the sphere that is farthest to the point (1, 2, 3). Hence, determine its distance.

**(i) Method 1**

When x = 1 and y = 10, , .

The points on the sphere are

For point , the normal of the plane is

Hence, the equation of the plane is

For point , the normal of the plane is

Hence, the equation of the plane is

**Method 2 This method can be used for any surface other than sphere**

Let

The equation of the tangent plane to the surface at is given by

Since

The equation of the tangent plane at is

or

The equation of the tangent plane at is

or

**(ii)**  The line joining the point (1, 2, 3) and the origin is

This line cuts the sphere, hence

When , .

Hence (-3, -6, -9) is a point on the sphere that is farthest to the point (1, 2, 3).

Its distance is

Note that this distance is also the sum of the distances from (1, 2, 3) to origin and the radius of the sphere =

**17/12/2017**

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